Quality-Dependent Stochastic Networks: Is FIFO Always Better Than LIFO?

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ABSTRACT

In various service and production systems, such as metal processing, fresh food industry, polymer forming, etc., the quality of the final product is a function of its total sojourn time in the system. We consider (i) an $M/G/1$-type single-server queue, as well as (ii) open tandem Jackson networks, and analyze the quality of products traversing through each system under two service disciplines â FIFO and LIFO. Although the mean sojourn times in an $M/G/1$ queue are equal under both service disciplines (that is, $E[W_{LIFO}] = E[W_{FIFO}]$), the variance of W_{LIFO} is larger than the variance of W_{FIFO}. We show that, when quality is the measure of performance, FIFO is not necessarily better than LIFO. We consider several service-time distributions and show under what values of the parameters one discipline is better (or worse) than the other. In particular, the mean quality under LIFO is better than the mean quality under FIFO for all values of the traffic intensity. Moreover, the mean quality under the FIFO service regime drops sharply to 0 when the traffic intensity approaches 1, while the mean quality under LIFO is bounded below. Numerical results as functions of the system parameters are presented and discussed for both the $M/G/1$ queue and the tandem Jackson network.

CCS CONCEPTS

• **Mathematics of computing** → **Queueing theory**; • **Networks** → **Network performance evaluation**.

KEYWORDS

Quality, FIFO, LIFO $M/G/1$,Tandem Jackson Network

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1 INTRODUCTION

Queueing theory deals primarily with probabilistic analyses of various types of service systems under a variety of probabilistic assumptions on the processes driving such systems. Usually the study is directed towards analyzing the distribution of the number of customers in the system and of their sojourn times, as well as finding the corresponding means and variances. In many production systems, e.g. fresh food industry, metal processing, polymer forming, etc., the quality of products traversing through the system deteriorates along with their sojourn time (see e.g. Liberopoulos et al, 2007 [18]). In such cases it is essential to determine the order of service of products, namely, serving them according to the First In First Out (FIFO) regime, or alternatively according to the Last In Last Out (LIFO) regime (Other services regimes, such as Random Order Of Service or Processor Sharing are not discussed in the current study). Also, it is important to determine the best order in which the product passes through a network of service sites, and as a result improve the product's quality and thus its value and reputation (see, e.g., a survey by Inman et al., 2013 [9]).

Deterioration of products is treated extensively in the manufacturing and inventory management literature, and it often aims at calculating the level of inventory stock and necessary lead time in purpose of minimizing costs and avoiding shortage (see e.g. Van Horenbeek et al., 2013 [25]). The inventory's quality analysis mainly deals with defective items due to imperfect manufacturing process, or damages (see e,g. see Kim and Gershwin, 2005 [14]). In addition, there is an abundance of research over quality-dependent products, where the majority of works deal with perishable products, with quality defined as a time-based dichotomous function (see e.g., Cooper, 2001 [6]; Berk and Gurler, 2008 [2]; Liberopoulos et al., 2007 [19]; Avinadav and Arponen, 2009 [1]). Naebulharam and Zhang (2013) [20] studied a Bernoulli quality model which determines the quality of each part, defective or non-defective, by a series of i.i.d. Bernoulli random variables. Coledani et al. (2015) [5] considered a discrete-time discrete-state Markov chain to examine the effects of buffer sizes on the probability that a product is defective or not in a multi-stage production system. Bortolini et al. (2016) [3] described a piecewise linear function between quality and time, and used it to estimate the market purchase probability. Goyal and Giri (2001) [8] and Perlman and Yechiali (2019) [21] assumed an exponential decay function of the product's quality. Our model also follows the exponential decay assumption and quantitatively defines the quality of a product passing through a production (or service) system.

The sojourn time of a product depends on various system characteristics, such as job-arrival process, distribution of service (processing) times, inner order of service and the sequence of service stations in case of a tandem network. Studies on manufacturing systems examined the effect of system parameters on productivity efficiency and production costs. Regarding the single server model, Levi and Yechiali (1975) [16] indicated the decomposition phenomenon of waiting times for the $M/G/1$ queue with multiple vacation, and derived the corresponding Laplace-Stieltjes Transform (LST). Scholl and Kleinrock [23] investigated the multiple vacation $M/G/1$ queue and derived the LSTs of waiting times under the FIFO, LIFO and ROS (Random Order of Service) regimes. Rosenberg and Yechiali (1993) [22] studied the batch arrival $M^x/G/1$ queue with single and with multiple vacations and derived explicit formulas for the LSTs, means and second moments of the waiting time for both FIFO and LIFO service-order regimes. They showed that although $E[W_{FIFO}] = E[W_{LIFO}]$, the second moment under LIFO is larger than that under the FIFO, and gave an explicit formula, $E\big[(W^q_{FIFO})^2\big] = E\big[(W^q_{LIFO})^2\big]/(1-\rho)$, for all cases. The current study relies on those results to compute the means and variances of a product's quality for the $M/G/1$ queue under both LIFO and FIFO service regimes.

Research over a network of queues has began with the seminal works of R.R.P. Jackson (1954 [12]; 1956 [13]) and J.R Jackson (1957 [10]; 1963 [11]). They developed a Markovian model, called Jackson network, and characterized the transition probabilities and basic system parameters of such networks. In a further work by Dallery and Frein (1993) [7] a manufacturing system was investigated as an open tandem Jackson network with either finite or infinite buffer between sites. They showed how using the decomposition phenomena allows the computation of basic system performance measures. Other researches (see Yechiali, 1988 [27]; Brandon and Yechiali, 1991 [4]) studied the n-node tandem queueing network with Bernoulli feedback to the first node and derived the LST of the total sojourn time of a product in the system. Perlman and Yechiali (2019) [21] investigated an n-site tandem network where each site consists of two stages: a processing stage and an inspection stage. The processing operation is a generally distributed random duration which either does or does not conclude successfully; in the latter case, the operation is repeated immediately. Once the processing stage concludes successfully, the product goes through an inspection stage which determines whether it will (i) require an additional processing and move forward to the next station; or (ii) it is found 'good' and exits the system with quality value depending on its sojourn time; or (iii) it is declared as 'failed' and exits the system with zero quality value. In their study they derived explicit results for the mean sojourn time of a product in the system, and its mean quality while assuming that each site is comprised of two $M/M/1$-type independent queues with FIFO service regime. In our work we expand this model and give an explicit formula for a tandem Jackson network where each site is an $M/M/1$ queue with LIFO service regime, and compare the results between the FIFO and the LIFO disciplines.

This paper treats two basic models (i) an $M/G/1$-type single-server queue, and (ii) an open tandem Jackson network. The objective is to analyze the quality of products traversing through each system under two service disciplines — FIFO and LIFO. In Section 2 the $M/G/1$ model is presented, and formulas are derived for the quality's mean and variance for both FIFO and LIFO service regimes. Two service time distributions are investigated: Exponential, and two-phase Erlang. The means and variances under each service time distribution are compared. It is shown that FIFO is not necessarily better then LIFO, as probably expected, but rather, the mean quality under LIFO is better then the mean quality under FIFO for all values of the traffic intensity. In Section 3 the tandem Jackson network is analyzed and formulas are derived for the mean quality of a product passing through the system, under both the FIFO and LIFO service regimes. Numerical results further show the relations between the two service disciplines. Conclusion are discussed in Section 4.

2 MODEL FORMULATION - SINGLE SERVER

We consider an M/G/1 queueing system where a Poisson stream of jobs (customers, calls, particles, products, etc.), with rate λ, flows into a single-server station. In steady state, let L denote the number of jobs in the system, and let $P_n = P(L = n)$, $n = 0, 1, \ldots$. Service time, B, of an arbitrary job is generally distributed with finite mean $E[B]$, finite second moment $E[B^2]$, and Laplace Stieltjes Transform (LST) $\widetilde{B}(s)$. Let W^q denote the queueing time (not including service) of a job, and let $W = W^q + B$ denote its overall sojourn time. Let $\rho = \lambda E[B] < 1$.

Consider the FIFO (First In First Out) service regime. The LST of the queueing time of an arbitrary job is given by (see Kleinrock (1975) [15], p. 199)

$$\widetilde{W^q}_{FIFO}(s) = \frac{s(1-\rho)}{[s-\lambda+\lambda\widetilde{B}(s)]} \tag{1}$$

which is known as the Pollaczek—Khinchine formula. The mean queueing time, $E[W^q_{FIFO}]$, is given by

$$E[W^q_{FIFO}] = \frac{\lambda E[B^2]}{2(1-\rho)}. \tag{2}$$

The LST of the sojourn time of a job is given by

$$\widetilde{W}_{FIFO}(s) = \widetilde{W^q}_{FIFO}(s)\widetilde{B}(s) = \frac{s(1-\rho)\widetilde{B}(s)}{[s-\lambda+\lambda\widetilde{B}(s)]}. \tag{3}$$

Let θ denote the length of a busy period, which is the time from the first arrival to an empty system until the system is empty again for the first time. The mean and LST of θ are the solution for the following equations (see Kleinrock (1975) [15], p. 212): $E[\theta] = \frac{E[B]}{1-\rho}$, and

$$\widetilde{\theta}(s) = \widetilde{B}[s+\lambda-\lambda\widetilde{\theta}(s)]. \tag{4}$$

Now, consider the non-preemptive LIFO (Last In Fast Out) service discipline where the server serves each job completely, and immediately afterwards begins to serve the last job to have arrived (if any). The LST and mean of the queueing time of an arbitrary job are given (see Rosenberg and Yechiali 1993 [22]) by

$$\widetilde{W^q}_{LIFO}(s) = (1-\rho) + \frac{\lambda\big[1-\widetilde{\theta}(s)\big]}{s+\lambda\big[1-\widetilde{\theta}(s)\big]}, \tag{5}$$

and

$$E[W^q_{LIFO}] = \frac{\lambda E[B^2]}{2(1-\rho)} = E[W^q_{FIFO}]. \tag{6}$$

The LST of the sojourn time W_{LIFO} is given by

$$\widetilde{W}_{LIFO}(s)=\widetilde{W}^{q}_{LIFO}(s)\widetilde{B}(s)=(1-\rho)\widetilde{B}(s)+\frac{\lambda\big[1-\widetilde{\theta}(s)\big]\widetilde{B}(s)}{s+\lambda\big[1-\widetilde{\theta}(s)\big]}. \quad (7)$$

It follows (see Rosenberg and Yechiali (1993)[22]), that although $E[W^q_{FIFO}]=E[W^q_{LIFO}]$, the second moments are related by $E[(W^q_{LIFO})^2]=\frac{1}{1-\rho}E[(W^q_{FIFO})^2]$.

2.1 Quality analysis for $M/G/1$ queue

While waiting in line, a product suffers degradation in its quality. The quality of a product that sojourns W units of time is considered here as $Q=\delta\, e^{-\gamma W}$, where δ is the potential quality in case $W=0$, and γ is a positive constant serving as a scaling factor. This formulation follows other works dealing with product degradation (see, e.g. Perlaman and Yachiali (2019) [21] and references there). For the $M/G/1$ queue, the expected value of a product's quality under the FIFO service regime is given by (see Eq. (3))

$$E[Q_{FIFO}]=E[\delta e^{-\gamma W_{FIFO}}]=\delta\widetilde{W}_{FIFO}(\gamma)=\delta\frac{\gamma(1-\rho)\widetilde{B}(\gamma)}{\gamma-\lambda+\lambda\widetilde{B}(\gamma)}. \quad (8)$$

Clearly, when $\rho\to 1$, $E[W_{FIFO}]\to\infty$ and $E[Q_{FIFO}]\to 0$. However, when $\lambda\to 0$ $(\rho\to 0)$, $E[Q_{FIFO}]\to\delta\widetilde{B}(\gamma)$. Indeed, in the latter case, upon arrival, every product finds an empty system so that its sojourn time is only its service duration B. Thus, $E[Q_{FIFO}]=E[-\delta e^{-\gamma B}]=\delta\widetilde{B}(\gamma)$.

The second moment of Q_{FIFO} can be calculated by using Eq. (8):

$$E[Q^2_{FIFO}]=E[\delta^2 e^{-2\gamma W_{FIFO}}]=\delta^2\widetilde{W}_{FIFO}(2\gamma)=\delta^2\frac{2\gamma(1-\rho)\widetilde{B}(2\gamma)}{2\gamma-\lambda+\lambda\widetilde{B}(2\gamma)}. \quad (9)$$

The Variance of Q_{FIFO} is calculated by

$$V[Q_{FIFO}]=E[Q^2_{FIFO}]-E^2[Q_{FIFO}]=\delta^2\frac{2\gamma(1-\rho)\widetilde{B}(2\gamma)}{2\gamma-\lambda+\lambda\widetilde{B}(2\gamma)}-\left[\delta\frac{\gamma(1-\rho)\widetilde{B}(\gamma)}{\gamma-\lambda+\lambda\widetilde{B}(\gamma)}\right]^2. \quad (10)$$

Now, when $\rho\to 1$, $V[Q_{FIFO}]\to 0$, while when $\lambda\to 0$ $(\rho\to 0)$,

$$V[Q_{FIFO}]\to\delta^2\widetilde{B}(2\gamma)-\delta^2\widetilde{B}^2(\gamma). \quad (11)$$

Regarding the LIFO service discipline, by using Eq. (7) the mean quality of a product is calculated as

$$E[Q_{LIFO}]=\delta E[e^{-\gamma W_{LIFO}}]=\delta\widetilde{W}_{LIFO}(\gamma)=\delta\left[(1-\rho)\widetilde{B}(\gamma)+\frac{\lambda[1-\widetilde{\theta}(\gamma)]\widetilde{B}(\gamma)}{\gamma+\lambda\big[1-\widetilde{\theta}(\gamma)\big]}\right]. \quad (12)$$

In this case too, $\lim_{\lambda\to 0}E[Q_{LIFO}]=\delta\widetilde{B}(\gamma)$. However, when $\rho\to 1$ (i.e. $\lambda\to\frac{1}{E[B]}$),

$$\lim_{\rho\to 1}E[Q_{LIFO}]=\frac{[1-\widetilde{\theta}(\gamma)]\widetilde{B}(\gamma)}{E[B]\gamma+[1-\widetilde{\theta}(\gamma)]}. \quad (13)$$

The second moment of Q_{LIFO} is calculated as

$$E[Q^2_{LIFO}]=E[\delta^2 e^{-2\gamma W_{LIFO}}]=\delta^2E[e^{-2\gamma W_{LIFO}}]=\delta^2\widetilde{W}_{LIFO}(2\gamma)=\delta^2(1-\rho)\widetilde{B}(2\gamma)+\delta^2\frac{\lambda[1-\widetilde{\theta}(2\gamma)\widetilde{B}(2\gamma)}{2\gamma+\lambda[1-\widetilde{\theta}(2\gamma)]}. \quad (14)$$

The Variance of Q_{LIFO} can be computed by using Eq.(12) and (14):

$$V[Q_{LIFO}]=E[Q^2_{LIFO}]-E^2[Q_{LIFO}]=\delta^2(1-\rho)\widetilde{B}(2\gamma)+\delta^2\frac{\lambda[1-\widetilde{\theta}(2\gamma)]\widetilde{B}(2\gamma)}{2\gamma+\lambda[1-\widetilde{\theta}(2\gamma)]}-\delta^2\left[(1-\rho)\widetilde{B}(\gamma)+\frac{\lambda[1-\widetilde{\theta}(\gamma)]\widetilde{B}(\gamma)}{\gamma+\lambda\big[1-\widetilde{\theta}(\gamma)\big]}\right]^2. \quad (15)$$

For the LIFO case, when $\rho\to 1$ (i.e. $\lambda\to\frac{1}{E[B]}$),

$$\lim_{\rho\to 1}V[Q_{LIFO}]=\delta^2\lim_{\rho\to 1}\left[\frac{[1-\widetilde{\theta}(2\gamma)]\widetilde{B}(2\gamma)}{E[B]2\gamma+[1-\widetilde{\theta}(2\gamma)]}-\left(\frac{[1-\widetilde{\theta}(\gamma)]\widetilde{B}(\gamma)}{E[B]\gamma+[1-\widetilde{\theta}(\gamma)]}\right)^2\right]. \quad (16)$$

For $\lambda\to 0$, as in the FIFO case, from Eq. (15),

$$V[Q_{LIFO}]\to\delta^2\widetilde{B}(2\gamma)-\delta^2\widetilde{B}^2(\gamma). \quad (17)$$

In Sections 2.2, and 2.3 we consider two service distributions, namely, (i) Exponential and (ii) two-phase Erlang.

2.2 Quality analysis for Exponential service times

2.2.1 FIFO Service Regime. For the $M(\lambda)/M(\mu)/1$ queue, where $B\sim Exp(\mu)$ and $\widetilde{B}(s)=\frac{\mu}{\mu+s}$, the sojourn time of a product under the FIFO service regime, W_{FIFO}, is distributed Exponentially with parameter $\mu-\lambda$. Thus,

$$\widetilde{W}_{FIFO}(s)=\frac{(\mu-\lambda)}{(\mu-\lambda)+s}. \quad (18)$$

Then, the expected quality of a product is given by Eqs. (8) and (18):

$$E[Q_{FIFO}]=\delta\widetilde{W}_{FIFO}(\gamma)=\delta\frac{(\mu-\lambda)}{(\mu-\lambda)+\gamma}. \quad (19)$$

As in Section 2.1, when $\lambda\to 0$, $E[Q_{FIFO}]\to\delta\frac{\mu}{\mu+\gamma}=\delta\widetilde{B}(\gamma)$, whereas when $\rho\to 1$ (i.e. $\lambda\to\mu$), $E[Q_{FIFO}]\to 0$.

The Variance of Q_{FIFO} is calculated by using Eq. (10):

$$V[Q_{FIFO}]=\delta^2(\mu-\lambda)\left[\frac{1}{\mu+2\gamma-\lambda}-\frac{\mu-\lambda}{(\mu+\gamma-\lambda)^2}\right]. \quad (20)$$

It follows that, as in Section 2.1, when $\lambda\to\mu$ (i.e. $\rho\to 1$), $V[Q_{FIFO}]\to 0$ (see also Fig. 1), while when $\lambda\to 0$,

$$V[Q_{FIFO}]\to\delta^2\mu\left[\frac{1}{\mu+2\gamma}-\frac{\mu}{(\mu+\gamma)^2}\right]=\delta^2\big[\widetilde{B}(2\gamma)-\widetilde{B}(\gamma)^2\big]. \quad (21)$$

2.2.2 *LIFO Service Regime.*

For the LIFO service discipline in an $M(\lambda)/M(\mu)/1$ queue, where $\widetilde{B}(\gamma) = \frac{\mu}{\mu+\gamma}$, we get by Eq. (12):

$$E[Q_{LIFO}] = \frac{\delta(\mu-\lambda)}{\mu+\gamma} + \frac{\delta\lambda\mu[1-\widetilde{\theta}(\gamma)]}{(\mu+\gamma)(\gamma+\lambda[1-\widetilde{\theta}(\gamma)])}. \quad (22)$$

Note that the length of the busy period in a work conserving system is independent of the service regime. Thus, by using Eq. (4), we obtain

$$\widetilde{\theta}(\gamma) = \widetilde{B}(\gamma+\lambda-\lambda\widetilde{\theta}(\gamma)) = \frac{\mu}{\mu+[\gamma+\lambda-\lambda\widetilde{\theta}(\gamma)]}, \quad (23)$$

resulting in

$$\widetilde{\theta}(\gamma) = \frac{\gamma+\lambda+\mu-\sqrt{(\gamma+\lambda+\mu)^2-4\lambda\mu}}{2\lambda}. \quad (24)$$

By substituting Eq. (24) in Eq. (22) we get an explicit expression for $E[Q_{LIFO}]$. That is,

$$E[Q_{LIFO}] = \frac{\delta}{\mu+\gamma}\left[2\mu-\lambda-\frac{2\gamma\mu}{\lambda+\gamma-\mu+\sqrt{(\gamma+\lambda+\mu)^2-4\lambda\mu}}\right]. \quad (25)$$

Indeed, as in Section 2.1, when $\lambda \to 0$, $E[Q_{LIFO}] = \delta\frac{\mu}{\mu+\gamma} = \delta\widetilde{B}(\gamma)$, whereas when $\rho \to 1$ (i.e. $\lambda \to \mu$)

$$E[Q_{LIFO}] \to \frac{\delta}{\mu+\gamma}\left[\mu-\frac{2\gamma\mu}{\gamma+\sqrt{\gamma^2+4\gamma\mu}}\right]. \quad (26)$$

The variance of Q_{LIFO}, when substituting $\widetilde{B}(2\gamma) = \frac{\mu}{\mu+2\gamma}$ in Eq. (15), results in

$$V[Q_{LIFO}] = \delta^2\frac{(\mu-\lambda)}{(\mu+2\gamma)} + \delta^2\frac{\lambda\mu[1-\widetilde{\theta}(2\gamma)]}{(\mu+2\gamma)(2\gamma+\lambda[1-\widetilde{\theta}(2\gamma)])} - \frac{\delta^2}{(\mu+\gamma)^2}\left[(\mu-\lambda)^2+\frac{2\lambda\mu(\mu-\lambda)[1-\widetilde{\theta}(\gamma)]}{\gamma+\lambda[1-\widetilde{\theta}(\gamma)]} + \left[\frac{\lambda\mu[1-\widetilde{\theta}(\gamma)]}{\gamma+\lambda[1-\widetilde{\theta}(\gamma)]}\right]^2\right], \quad (27)$$

where

$$\widetilde{\theta}(2\gamma) = \frac{2\gamma+\lambda+\mu-\sqrt{(2\gamma+\lambda+\mu)^2-4\lambda\mu}}{2\lambda}. \quad (28)$$

2.2.3 *Comparison between LIFO and FIFO for $M(\lambda)/M(\mu)/1$ queue.*

In this section the mean qualities and quality variances under the LIFO and under the FIFO service regimes are compared for the $M/M/1$ queue. Using Eqs. (19) and (25), the ratio between the two mean qualities is given by

$$\frac{E[Q_{LIFO}]}{E[Q_{FIFO}]} = \frac{(\mu-\lambda+\gamma)}{(\mu-\lambda)(\mu+\gamma)}\left[2\mu-\lambda-\frac{2\gamma\mu}{\lambda+\gamma-\mu+\sqrt{(\gamma+\lambda+\mu)^2-4\lambda\mu}}\right]. \quad (29)$$

Propositions 1 and 2 below show the advantage of the LIFO over the FIFO with respect to mean qualities.

Proposition 2.1. *1 For $\mu, \gamma > 0$ and $0 < \lambda \leq \mu$,*

$$\lim_{\lambda\to\mu}\frac{E[Q_{LIFO}]}{E[Q_{FIFO}]} = \infty.$$

Proof. The claim follows directly from Eq. (29). □

Proposition 2.2. *2 For $\gamma > 0$ and $0 < \lambda < \mu$,*

$$\frac{E[Q_{LIFO}]}{E[Q_{FIFO}]} > 1.$$

Proof. By Eq. (29) $\frac{E[Q_{LIFO}]}{E[Q_{FIFO}]} > 1 \iff$

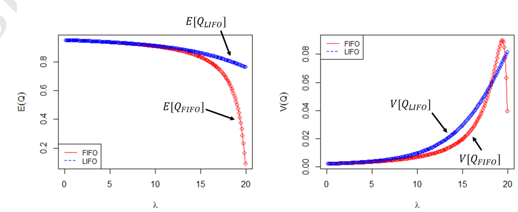
$$(\mu-\lambda+\gamma)(2\mu-\lambda)\left[\lambda+\gamma-\mu+\sqrt{(\gamma+\lambda+\mu)^2-4\lambda\mu}\right]+2\gamma\mu > (\mu-\lambda)(\mu+\gamma)\left[\lambda+\gamma-\mu+\sqrt{(\gamma+\lambda+\mu)^2-4\lambda\mu}\right]. \quad (30)$$

Let $C := \lambda+\gamma-\mu+\sqrt{(\gamma+\lambda+\mu)^2-4\lambda\mu}$ denote the the numerator of $\widetilde{\theta}(\gamma)$ in Eq. (24). Clearly, $C > 0$. Substituting C in Eq. (30) translates to

$$(\mu-\lambda)(2\mu-\lambda)C+\gamma(2\mu-\lambda)C+2\gamma\mu > (\mu-\lambda)(\mu+\gamma)C \iff (\mu-\lambda)^2C+\gamma\mu C+2\gamma\mu > 0, \quad (31)$$

which holds since $\gamma\mu > 0$ and $(\mu-\lambda)^2 > 0$. □

Figure 1 depicts the behaviour of the mean qualities (Figure 1(a)) and variances (Figure 1(b)) for both FIFO (red) and LIFO (blue) service regimes as functions of λ when $\mu = 20$, $\gamma = 1$, $\delta = 1$. $E[Q_{LIFO}]$, is a monotonously decreasing concave function of λ, approaching 0.76 as $\lambda \to \mu$ (see Eq. (26)). $E[Q_{FIFO}]$ is also concave, but in contrast to $E[Q_{LIFO}]$, it decreases sharply when λ gets closer to μ, reaching 0 as $\lambda = \mu$. Figure 1(a) illustrates graphically the claims of proposition 1 and 2.



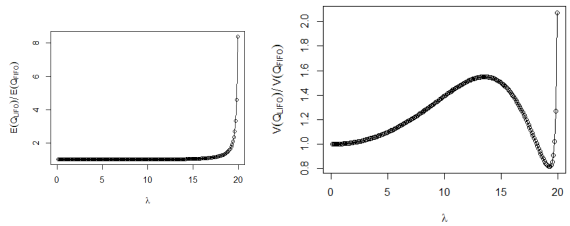
(a) $E[Q]$ as a function of λ (b) $V[Q]$ as a function of λ

Figure 1: Mean and variance of a product's quality under FIFO and under LIFO service regimes for Exponential service times as a function of λ, when $\mu = 20$, $\gamma = 1$, $\delta = 1$

Figure 1(b) depicts the behaviour of the variances of the two service regimes as functions of λ for the same parameter values as in figure 1(a). It is seen that both variances increase when λ is small or moderate. As long as λ is smaller then 18.2, $V(Q_{FIFO}) < V(Q_{LIFO})$, but when λ exceeds 18.2, the ratio turns over and $V(Q_{FIFO}) > V(Q_{LIFO})$. As λ further increases $V(Q_{FIFO})$ changes its direction at its highest value $V(Q_{FIFO}) = 0.09$, and then drops sharply towards 0, crossing $V(Q_{LIFO})$ at $\lambda = 19.6$.

The above observations are explained as follows: When λ is small, there is a significant probability under the LIFO regime that the last arriving product will be admitted to service before another arrival occurs, while if not, the product will have to wait at least one busy period. These two opposite possibilities increase the value of the quality's variance under FIFO regime. When λ approaches μ, the queue becomes very large and every product waits a very long time, so that $V[Q_{FIFO}]$ decreases sharply.

Figures 2 further depicts the ratios between the mean qualities of the two regimes and the ratio between their variances. In figure 2(a) it is seen that the ratio increases slowly up to $\lambda = 18$ and then jumps to ∞. Figure 2(b) illustrates that the ratio between the variances increases up to $\lambda = 13.5$, then decreases sharply up to $\lambda = 19.2$, and then jumps almost instantly towards ∞. These turning points in the ration coincides with the two crossing points of Figure 1(b).



(a) Ratio between Expected quality in LIFO regime vs FIFO regime

(b) Ratio between the quality Variances of the two regimes

Figure 2: Comparing quality means and variances for FIFO and LIFO service regimes for Exponential service times, when $\mu = 20$, $\gamma = 1$, $\delta = 1$

2.3 Quality analysis for two-phase Erlang service times

In this section we consider an $M/G/1$ queue, with Poisson arrival rate λ and Erlang service time having two consecutive exponential (μ) durations. That is, $B \sim Eralng(2, \mu)$. We have $\widetilde{B}(s) = (\frac{\mu}{\mu+s})^2$, $\rho = \frac{2\lambda}{\mu}$. The stability condition $\rho < 1$ translates here to $\lambda < \frac{\mu}{2}$.

2.3.1 FIFO Service Regime in $\mathbf{M(\lambda)/E(2,\mu)/1}$ queue. Under the FIFO regime, by using Eq. (8), the mean quality of a departing product is given by

$$E[Q_{FIFO}] = \delta \frac{\mu(\mu - 2\lambda)}{(\mu + \gamma)^2 - \lambda(2\mu + \gamma)}. \tag{32}$$

By using Eq. (10), the variance of Q_{FIFO} is given by

$$V[Q_{FIFO}] = \delta^2 \frac{2\gamma\mu(\mu - 2\lambda)}{(2\gamma - \lambda)(\mu + 2\gamma)^2 + \lambda\mu^2} - \left[\delta \frac{\gamma\mu(\mu - 2\lambda)}{(\gamma - \lambda)(\mu + \gamma)^2 + \lambda\mu^2}\right]^2. \tag{33}$$

2.3.2 LIFO Service Regime in $\mathbf{M(\lambda)/E(2,\mu)/1}$ queue. Under LIFO, by using Eq. (12) the mean quality is given by

$$E[Q_{LIFO}] = \delta \frac{\mu(\mu - 2\lambda)}{(\mu + \gamma)^2} + \delta \frac{\lambda\mu^2[1 - \widetilde{\theta}(\gamma)]}{(\mu + \gamma)^2(\gamma + \lambda[1 - \widetilde{Q}(\gamma)])}, \tag{34}$$

where here $\widetilde{\theta}(\gamma)$ can be calculated by Eq. (4):

$$\widetilde{\theta}(\gamma) = \widetilde{B}(\gamma + \lambda - \lambda\widetilde{\theta}(\gamma)) = \frac{\mu^2}{\left[\mu + \left[\gamma + \lambda - \lambda\widetilde{\theta}(\gamma)\right]\right]^2} \tag{35}$$

This leads to a cubic equation in $\widetilde{\theta}(\gamma)$:

$$\lambda^2\widetilde{\theta}^3(\gamma) - 2\lambda(\mu + \gamma + \lambda)\widetilde{\theta}^2(\gamma) + (\mu + \gamma + \lambda)^2\widetilde{\theta}(\gamma) - \mu^2 = 0. \tag{36}$$

Denote the coefficients in the 3rd degree polynomial given in Eq. (36) as $a = \lambda^2$, $b = -2\lambda(\mu + \gamma + \lambda)$, $c = (\mu + \gamma + \lambda)^2$, and $d = -\mu^2$.

Then, by using Cardano's Formula for solving a cubic equation (see Witula and Slota, 2009 [26]) we get three roots, only one of them is real and positive, so it is the only valid candidate for $\widetilde{\theta}(\gamma)$:

$$\widetilde{\theta}(\gamma) = S + T + \frac{2(\gamma + \mu + \lambda)}{3\lambda}, \tag{37}$$

where

$$S = \sqrt[3]{R + \sqrt{P^3 + R^2}} \qquad T = \sqrt[3]{R - \sqrt{P^3 + R^2}}, \tag{38}$$

while,

$$\begin{aligned} R &= \frac{9abc - 27a^2d - 2b^3}{54a^3} = \frac{2(\gamma + \mu + \lambda)^3 + 27\lambda\mu^2}{54\lambda^3} \\ P &= \frac{3ac - b^2}{9a^2} = -\frac{(\gamma + \mu + \lambda)^2}{9\lambda^2} \end{aligned} \tag{39}$$

$E[Q_{LIFO}]$ is calculated for the $M(\lambda)/E(2, \mu)/1$ queue by substituting first Eq. (39) in Eq. (38), then Eq. (38) in Eq (37), and finally Eq. (37) in Eq. (34).

The Variance of Q_{LIFO} is given by

$$\begin{aligned} V[Q_{LIFO}] =& \delta^2 \frac{\mu(\mu - 2\lambda)}{(\mu + 2\gamma)^2} + \delta^2 \frac{\lambda\mu^2[1 - \widetilde{\theta}(2\gamma)]}{(\mu + 2\gamma)^2(2\gamma + \lambda[1 - \widetilde{\theta}(2\gamma)])} - \\ & \delta^2 \frac{\mu^2(\mu - 2\lambda)^2}{(\mu + \gamma)^4} + 2\delta^2 \frac{\lambda\mu^3(\mu - 2\lambda)[1 - \widetilde{\theta}(\gamma)]}{(\mu + \gamma)^4(\gamma + \lambda[1 - \widetilde{\theta}(2\gamma)])} - \\ & \delta^2 \frac{\lambda^2\mu^4[1 - \widetilde{\theta}(\gamma)]^2}{(\mu + \gamma)^4(\gamma + \lambda[1 - \widetilde{\theta}(2\gamma)])^2}, \end{aligned} \tag{40}$$

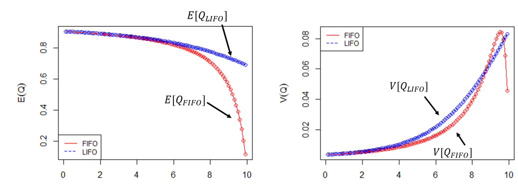
where $\widetilde{\theta}(2\gamma)$ is the real positive root of the following cubic equation:

$$\lambda^2\widetilde{\theta}^3(2\gamma) - 2\lambda(\mu + 2\gamma + \lambda)\widetilde{\theta}^2(2\gamma) + (\mu + 2\gamma + \lambda)^2\widetilde{\theta}(2\gamma) - \mu^2 = 0 \tag{41}$$

which is solved by Cardano's formula.

2.3.3 Comparison between LIFO and FIFO in $\mathbf{M(\lambda)/E(2,\mu)/1}$ queue. In this sub-section the FIFO and LIFO service regimes are compared numerically with respect to the $M/E_2/1$ queue.

Figures 3 bellow depicts the results for the mean qualities (Figure 3(a)) and variances (Figure 3(b)) for both FIFO (red) and LIFO (blue) regimes as functions of λ, when $\mu = 20$, $\gamma = 1$, $\delta = 1$. $E[Q_{LIFO}]$ is a concave monotonic decreasing function of λ, and approaches 0.7 as $\rho \to 1$, i.e., $\lambda \to \frac{\mu}{2} = 10$. In comparison, $E[Q_{FIFO}]$ is also concave, but decreases sharply when ρ gets closer to 1, reaching 0 at $\lambda = 10$. Note that $E[Q_{LIFO}]/E[Q_{FIFO}] > 1$ for all λ.

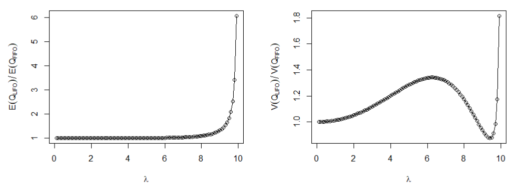


(a) $E[Q]$ as a function of λ **(b) $V[Q]$ as a function of λ**

Figure 3: Mean and variance of a product's quality under FIFO and under LIFO service regimes for two-phase Erlang service times ($E(\mu, 2)$), as functions of λ, when $\mu = 20$, $\gamma = 1$, $\delta = 1$

Figure 3(b) depicts the behaviour of the variances of the two service regimes as a function of λ for the same parameter values as in Figure 3(a). It is seen that both variances increase when λ is small or moderate. When λ is smaller than 8.8, $V[Q_{FIFO}] < V[Q_{LIFO}]$, but when λ exceeds $\lambda = 8.8$, the ratio turns over and $V[Q_{LIFO}] < V[Q_{FIFO}]$. $V[Q_{FIFO}]$ reverses its direction at its highest value $V[Q_{FIFO}] = 0.084$ and drops sharply towards 0, crossing $V[Q_{LIFO}]$ at $\lambda = 9.7$.

Figure 4 depicts the ratios between the mean qualities of the two regimes and the ratio between their variances. In Figure 4(a) the ratio between the mean qualities increases slowly up to $\lambda = 8.5$ and then jumps rapidly to ∞. Figure 4(b) illustrates that the ratio between the variances increases up to $\lambda = 6.3$, then decreases sharply up to $\lambda = 9.4$ and finally jumps almost instantly towards ∞. Observe that Figures 4(a) and 4(b) are similar, respectively, to Figures 2(a) and 2(b).



(a) Ratio between mean qualities: $E[Q_{LIFO}]/E[Q_{FIFO}]$ **(b) Ratio between the quality Variances:** $E[Q_{LIFO}]/E[Q_{FIFO}]$

Figure 4: Comparing quality's means and variances for FIFO and LIFO service regime for $M(\lambda)/E(2,\mu)/1$ queue, as a function of λ, when $\mu = 20$, $\gamma = 1$, $\delta = 1$

3 TANDEM JACKSON NETWORK (TJN) WITH TIME-DEPENDENT QUALITY DETERIORATION

Consider an n-site stochastic tandem Jackson network (TJN) where products arrive to the first site at a Poisson rate λ, and then, after being processed and inspected at each site, move uni-directionally from site to site, as depicted in Figure 5 (see Perlman and Yechiali, 2019 [21]).

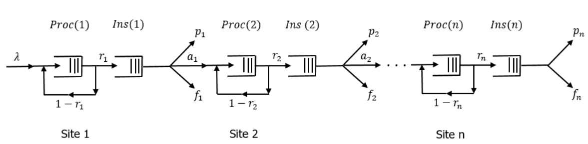


Figure 5: Tandem Stochastic Network. "Proc(i)" ("Ins(i)") indicates the processing (inspection) stage of site i

Each site consists of two separate and independent stages: a processing stage, followed by an inspection stage, where each stage is an $M/M/1$-type queue. A single processing attempt at stage i, B_i, is exponentially distributed with mean $1/\mu_i$ and LST $\widetilde{B_i}(s) = \frac{\mu_i}{s+\mu_i}$, and it concludes either successfully, with probability r_i, or fails with the complementary probability $1 - r_i$. In the latter case, the processing operation is immediately repeated with the same success probability, until a successful attempt. Thus, the total processing time of a product at site i, G_i, is a Geometric sum of iid Exponential random variables, and is distributed Exponentially with mean $E(G_i) = \frac{1}{\mu_i r_i}$ and LST given by $\widetilde{G}(s) = \frac{r_i\mu_i}{s+r_i\mu_i}$. It is well known (see Takagi, 1991 [24]) that the output process from an $M(\lambda)/M(*)/1$ queue is also Poisson with the same rate λ, and that the inter-departure times form a renewal process. Thus, each site can be treated as an isolated $M/M/1$ queue. Upon successful processing completion at site i, the product moves to the inspection stage of this site. The inspection time, D_i, is exponentially distributed with mean $1/\xi_i$ and LST $\widetilde{D_i}(s) = \frac{\xi_i}{s+\xi_i}$ $(i = 1, 2, \ldots, n)$.

Upon conclusion of the inspection, the state of the product is determined according to the following possibilities: (i) with probability a_i the product advances to queue $i+1$ $(1 \le i \le n-1)$; (ii) with probability p_i the product is declared to be 'good' and exits the system with a quality value Q depending on the total time it sojourned in the system; and (iii) with probability f_i the product is declared as 'failed', discarded and exits the system with quality 0. Clearly, $a_i + p_i + f_i = 1$ for every $i = 1, 2, \ldots, n$. Thus, the arrival rate to the processing stage at site i is $\lambda_i = \lambda \prod_{k=1}^{i-1} a_k$ for $1 < i \le n-1$, where $\prod_{k=1}^{0} a_k = 1$. Out of site n, $a_n = 0$, and $p_n + fn = 1$. The traffic load condition for the network to reach stability is $\lambda_i < min\{r_i\mu_i, \xi_i\}$ for each site $1 \le i \le n$.

During its traversal time through the system the product suffers degradation in its quality, depending on the total time it has sojourned in the system. Let T_j denote a product's accumulated traversal time through sites 1 to j. If the product exits the system at site j $(1 \le j \le n)$, its quality is either $Q = \delta e^{-\gamma T_j}$ with probability p_j , or zero with probability f_j . As indicated above, both stages are $M/M/1$ queues. Let W_i and V_i denote the total sojourn time in the processing stage, and in the inspection stage at site i, respectively. Then, the total traversal time of a product through sites 1 to j is $T_j = \sum_{i=1}^{j}(W_i + V_i)$.

3.1 FIFO Service Regime

The total sojourn time of a product in an $M(\lambda)/M(\mu)/1$ queue is exponentially distributed with mean $\frac{1}{\mu-\lambda}$. Thus, since the total processing time in site i is exponentially distributed with parameter $r_i\mu_i$, and the arrival rate is λ_i, under the FIFO regime, W_i^{FIFO} (a slight change of notation), the product's total processing time at site i, is exponentially distributed with parameter $r_i\mu_i - \lambda_i$ and LST $\widetilde{W}_i^{FIFO}(s) = \frac{r_i\mu_i-\lambda_i}{r_i\mu_i-\lambda_i+s}$.

Similarly, the inspection stage at site i is also an M/M/1 queue with arrival rate λ_i and inspection rate ξ_i. Consequently, the sojourn time V_i^{FIFO} of a product in the inspecting stage there is also exponentially distributed with parameter $\xi_i - \lambda_i$ and LST $\widetilde{V}_i^{FIFO}(s) = \frac{\xi_i-\lambda_i}{\xi_i-\lambda_i+s}$.

The mean traversal time in the system $E[T^{FIFO}]$ is given by (see Perlman and Yechiali [2019])

$$E[T^{FIFO}] = \sum_{j=1}^{n}\Bigg(\Big(\prod_{k=1}^{j-1} a_k\Big)(p_j + f_j)\sum_{i=1}^{j}\Big(\frac{1}{r_i\mu_i - \lambda_i} + \frac{1}{\xi_i - \lambda_i}\Big)\Bigg). \tag{42}$$

The mean traversal time for a product exiting the system as 'good' is given by (Perlman and Yechiali, 2019 [21])

$$E[T^{FIFO}|good] = \frac{\sum_{j=1}^{n}\Big(\big(\prod_{k=1}^{j-1} a_k\big)p_j \sum_{i=1}^{j}\big(\frac{1}{r_i\mu_i-\lambda_i}+\frac{1}{\xi_i-\lambda_i}\big)\Big)}{\sum_{j=1}^{n}\big(\prod_{k=1}^{j-1} a_k\big)p_j} \quad (43)$$

and the mean quality of a product leaving the system is given by

$$E[Q^{FIFO}] = \sum_{j=1}^{n}\Big(\big(\prod_{k=1}^{j-1} a_k\big)p_j\delta\prod_{i=1}^{j}\big(\frac{r_i\mu_i-\lambda_i}{r_i\mu_i-\lambda_i+\gamma}\cdot\frac{\xi_i-\lambda_i}{\xi_i-\lambda_i+\gamma}\big)\Big), \quad (44)$$

where $\lambda_i = \lambda\prod_{k=1}^{i-1} a_k$.

3.2 LIFO Service Regime

Considering site i, the LST of W_i^{LIFO} is given by Eq. (7). Thus, with $\rho = \frac{\lambda_i}{r_i\mu_i}$ and $\widetilde{G}_i(s) = \frac{r_i\mu_i}{r_i\mu_i+s}$, the LST of the the sojourn time in the *processing stage* is given by

$$\widetilde{W}_i^{LIFO}(s) = \frac{r_i\mu_i-\lambda_i}{r_i\mu_i+s} + \frac{\lambda_i r_i\mu_i\big[1-\widetilde{\theta}_i(s)\big]}{(r_i\mu_i+s)(s+\lambda_i\big[1-\widetilde{\theta}_i(s)\big])}, \quad (45)$$

where the LST of θ_i, the corresponding busy period, is given by

$$\widetilde{\theta}_i(s) = \frac{s+\lambda_i+r_i\mu_i-\sqrt{(s+\lambda_i+r_i\mu_i)^2-4\lambda_i r_i\mu}}{2\lambda_i}. \quad (46)$$

Similarly, the LST of the inspection time in site i, V_i^{LIFO}, is calculated with the aid of Eq. (7). Thus, with $\rho = \frac{\lambda_i}{\xi_i}$ and $\widetilde{D}_i(s) = \frac{\xi_i}{\xi_i+s}$, the LST of V_i^{LIFO} is given by

$$\widetilde{V}_i^{LIFO}(s) = \frac{\xi_i-\lambda_i}{\xi_i+s} + \frac{\lambda_i\xi_i\big[1-\widetilde{\phi}_i(s)\big]}{(\xi_i+s)(s+\lambda\big[1-\widetilde{\phi}_i(s)\big])}, \quad (47)$$

where $\widetilde{\phi}_i(s)$, denoting the busy period in the inspection stage at site i, is given by

$$\widetilde{\phi}_i(s) = \frac{s+\lambda_i+\xi_i-\sqrt{(s+\lambda_i+\xi_i)^2-4\lambda_i\xi_i}}{2\lambda_i}. \quad (48)$$

Thus, the mean quality of an arbitrary product exiting the system is given by:

$$E[Q^{LIFO}] = \sum_{i=1}^{n}\big(\prod_{k=1}^{i-1} a_k\big)p_i E[Q_i^{LIFO}], \quad (49)$$

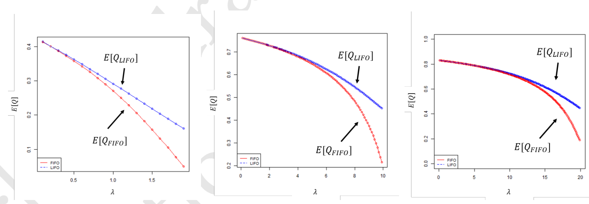
where $E[Q_i^{LIFO}]$, the mean quality of a product exiting the system in site i, is given by:

$$E[Q_i^{LIFO}] = E[^{-\gamma T_i^{LIFO}}] = \delta\widetilde{T}_i^{LIFO}(\gamma) = \delta\prod_{k=1}^{i}\widetilde{W}_i^{LIFO}(\gamma)\cdot\widetilde{V}_i^{LIFO}(\gamma)$$
$$= \delta\prod_{k=1}^{i}\Big(\frac{r_k\mu_k-\lambda_k}{r_k\mu_k+\gamma} + \frac{\lambda_k r_k\mu_k\big[1-\widetilde{\theta}_k(\gamma)\big]}{(r_k\mu_k+\gamma)(\gamma+\lambda_k\big[1-\widetilde{\theta}_i(\gamma)\big])}\Big)\times$$
$$\Big(\frac{\xi_k-\lambda_k}{\xi_k+\gamma} + \frac{\lambda_k\xi_k\big[1-\widetilde{\phi}_k(\gamma)\big]}{(\xi_k+\gamma)(\gamma+\lambda_k\big[1-\widetilde{\phi}_k(\gamma)\big])}\Big). \quad (50)$$

3.3 Comparison between LIFO and FIFO in TJN

In the following section the FIFO and LIFO service regimes are compared numerically for the tandem Jackson network with $n = 2$ sites. Two cases are investigated with various parameter values: (i) identical sites, and (ii) non-identical sites.

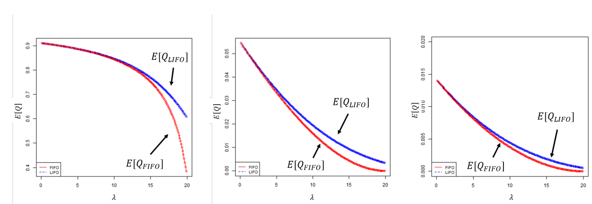
3.3.1 Identical sites (n=2). Assume that all processing stages are statistically identical and all inspection stages are also identical and compare mean quality as a function of λ. Figure 6 illustrates the results of $E[Q]$ for various values of r_i while $\gamma = 1$, $\delta = 1$, $\mu_i = 20$, $\xi_i = 20$. It is seen that. for all values of r_i, LIFO is better than FIFO for all values of λ. Under both service regimes, as r_i increases, the mean quality increases due to the decrease in sojourn time at every station. In addition, for $r_i = 0.1$ (Figure 6a) the shape of $E[Q_{LIFO}]$ is approximately linear while for $r_i = 0.5$ and $r_i = 1$ $E[Q_{LIFO}]$ is concave. In all cases $E[Q]$ decreases when λ increases.



(a) $E[Q]$ **for** $r_i = 0.1$ **(b)** $E[Q]$ **for** $r_i = 0.5$ **(c)** $E[Q]$ **for** $r_i = 1$

Figure 6: Mean product quality (FIFO and LIFO) in TJN with two nodes as a function of λ **for various values of** r_i **while** $\gamma = 1$, $\delta = 1$, $\mu_i = 20$, $\xi_i = 20$

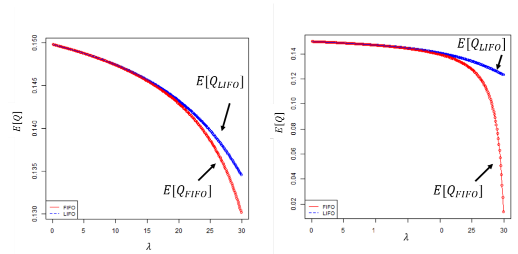
Figure 7 illustrates the results of $E[Q]$ as a function of λ for different values of the shape parameter γ, while $\delta = 1$, $r_i = 1$, $\mu_i = 20$, $\xi_i = 20$. It is seen that for all values of γ, LIFO is better than FIFO for all values of λ. When $\gamma = 0.5$, for both FIFO and LIFO, the mean quality is a concave function with increasing difference between FIFO and LIFO as λ approaches μ. In comparison, for $\gamma = 25$ and $\gamma = 50$, the mean quality under both regimes is convex, and the difference between FIFO and LIFO decreases as γ increases. This is due to the increasing deterioration rate as γ increases.



(a) $E[Q]$ **for** $\gamma = 0.5$ **as a function of** λ **(b)** $E[Q]$ **for** $\gamma = 25$ **as a function of** λ **(c)** $E[Q]$ **for** $\gamma = 50$ **as a function of** λ

Figure 7: Mean product quality for TJN (FIFO and LIFO) with two nodes for various values of γ **while** $\delta = 1$, $r_i = 1$, $\mu_i = 20$, $\xi_i = 20$

3.3.2 Non-identical processing sites. The parameters for the first site are: $r_1 = 1$, $p_1 = 0$, $a_1 = 0.8$ and $f_1 = 0.2$, while the parameters for the second site are: $r_2 = 1$, $p_2 = 0.2$, and $f_2 = 0.8$. Consider two cases: (i) $\mu_1 = 30$, $\mu_2 = 60$ and (ii) $\mu_1 = 60$, $\mu_2 = 30$. For simplicity we assume that inspection stages are identical with $\xi_1 = \xi_2 = 20$. Figure 8 bellow depicts the value for $E[Q]$ when $\delta = 1$, and $\gamma = 1$. Note that when the processing rate at site 1 is higher (Fig 8(a)), the deterioration as ρ approaches 1 in both service regimes is almost identical, while when the processing rate at site 1 is lower (Fig 8(b)), $E[Q^{FIFO}]$ is bounded (by 0.12) while $E[Q^{FIFO}]$ is decreasing sharply towards 0, as ρ approaches 1. Those results are explained by examining the results in previous sections regarding the $M/G/1$ queue: when ρ approaches 1, $E[Q_{FIFO}]$ deteriorates rapidly towards 0.



(a) $E[Q]$ for $\mu_1 = 60, \mu_2 = 30$ as a function of λ (b) $E[Q]$ for $\mu_1 = 30, \mu_2 = 60$ as a function of λ

Figure 8: Mean product quality in a two-node TJN (FIFO and LIFO) with non identical processing stages, while $\delta = 1$, $r_i = 1$, $\gamma = 1$, $\xi_i = 120$

4 CONCLUSIONS

This paper analyses the quality of a product traversing through (i) a single server $M/G/1$-type queue, and (ii) a tandem Jackson network. For the $M/G/1$ queue, two service time distributions are specifically studied - Exponential and two-phase Erlang (other service time distributions are discussed in a technical report by Liberman and Yechiali, 2019 [17]).

Assuming that the quality of a product deteriorates exponentially with the time it sojourns in the system, two service disciplines are compared - FIFO vs. LIFO (the investigation of other service regimes such as Random Order of Service or Processor Sharing are left to a further research). Interestingly, it is shown that FIFO is not necessarily better then LIFO. Numerical results show the behaviour of the quality mean, as well as the quality variance, as functions of the various system parameters. It is shown that the mean quality under LIFO is always greater then FIFO. The first is bounded below when the traffic intensity approaches 1, while the latter drops sharply to zero. Similar qualitative results hold for a tandem Jackson network. A further possible research may look at concave deteriorating quality function where the product is considered as perfectly good up to some fixed time threshold.

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